Abstracts of Invited Lectures

Title: Finite Element Differential Forms on Simplices and Cubes

Speaker: Douglas N. Arnold
Affiliation: University of Minnesota, USA
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Abstract: Many applications require finite element discretizations of the spaces of differential forms comprising the de Rham complex. In three dimensions, this means constructing finite element subspaces of $H^1$, $H(\text{curl})$, $H(\text{div})$, and $L^2$, with appropriate properties and interrelations. For simplicial meshes, there are two primary families of finite element differential forms, the $P_r \Lambda^k(T_h)$ and $P_{r-1} \Lambda^k(T_h)$ families, each defined for all polynomial degrees $r$, all form degrees $k$, and arbitrary simplicial meshes $T_h$ in any number of dimensions. In two and three dimensions, these families unify most of the known mixed finite element spaces for such problems. For cubic meshes, one family of finite element differential forms can be defined through a tensor product construction. This reduces to the well-known $Q_r$ finite elements in $H^1$ (the case of 0-forms), and recovers known mixed finite elements for higher order forms in two and three dimensions. The main new result we discuss is a recently discovered second family of finite element differential forms on cubical meshes. In the case of 0-forms this these are the serendipity elements, while for higher order forms they are new mixed finite element spaces for $H(\text{div})$ and $H(\text{curl})$.

Title: Control and Nash Games with Mean Field Effect

Speaker: Alain Bensoussan
Affiliation: University of Texas at Dallas, USA
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Abstract: Mean field theory has raised a lot of interest in the recent years, see in particular Lasry-Lions, Gueant-Lasry-Lions, Huang-Caines-Malham and many others. There are a lot of applications. In general the applications concern approximating an infinite number of players with common behavior by a representative agent. This agent has to solve a control problem perturbed by a field equation, representing in some way the behavior of the average infinite number of agents. This approach does not lead easily to problems of Nash equilibrium for a finite number of players, perturbed by field equations, unless one considers averaging within different groups, which has not been done in the literature, and seems quite challenging. In this paper, we come to similar problems, with a different motivation, which makes sense for control, but also for differential games. We thus consider systems of nonlinear partial differential equations with mean field terms, which has not been addressed in the literature so far.
Title: A Posteriori Analysis of the Time and Space Discretizations of Richards Equations

Speaker: Christine Bernardi
Affiliation: Université Pierre et Marie Curie, FRANCE
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Abstract: We consider the equation due to Richards which models the water flow in a partially saturated underground porous medium under the surface. We propose a discretization of this equation by an implicit Euler’s scheme in time and finite elements in space. We perform the a posteriori analysis of this discretization, in order to improve its efficiency via time step and mesh adaptivity. Some numerical experiments confirm the interest of this approach.

Title: Virtual Element Methods for Linear Elliptic Problems

Speaker: Franco Brezzi
Affiliation: Istituto di Matematica Applicatae Tecnologie Informatiche del CNR, ITALY
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Abstract: The Virtual Element Method is a new variant of Mimetic Finite Differences that allows a simpler theoretical treatment and a bigger generality. The new method retains the possibility to deal with very general geometries, but extends the MFD approach to higher order methods and, most important, to the use of $C^k$ approximations with $k$ bigger than zero. In particular for fourth order problems (like Kirchhoff-Love plates) the VEM are simpler and more effective than traditional FEM even on simple geometries, like triangles.

Title: A Class of Large Global Regular Solutions to the Incompressible Navier-Stokes Equations

Speaker: Jean-Yves Chemin
Affiliation: Université Pierre et Marie Curie, FRANCE
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Abstract: In this talk, we are going to recall the concept of large initial data based on the Koch and Tataru theorem for initial data in the space $BM0^{-1}$. Then, we are going to investigate the case of large initial data with slow variation in one direction. After a rescaling, we study an analogous of incompressible Navier-Stokes system with very small viscosity in one direction and with a perturbed pressure term which makes to the system (very probably) illposed. Then to bypass the difficulty, we introduced a global Cauchy-Kovalevska method which allows to solve global the rescaled system for small analytic initial data (which correspond to very large initial data in the original problem).
Title: Implicit Sampling, with Application to Data Assimilation

Speaker: Alexandre J. Chorin
Affiliation: University of California, USA
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Abstract: There are many computational tasks in which it is necessary to sample a given probability density (pdf), i.e., use a computer to construct a sequence of independent random vectors \(x_i, i = 1, 2, \ldots\), whose histogram converges to the given pdf. This can be difficult because the sample space can be huge, and more important, because the portion of the space where the density is significant can be very small, so that one may miss it by an ill-designed sampling scheme. Indeed, Markov-chain Monte Carlo, the most widely used sampling scheme, can be thought of as a search algorithm, where one starts at an arbitrary point and one advances step-by-step towards the high probability region of the space. This can be expensive, in particular because one is typically interested in independent samples, while the chain has a memory. We present an alternative, in which samples are found by solving an algebraic equation with a random right hand side rather than by following a chain; each sample is independent of the previous samples. We explain the construction in the context of numerical integration, and then apply it to data assimilation and to quantum Monte Carlo.

Title: Recent Advances in Linear and Nonlinear Shell Theories

Speaker: Philippe G. Ciarlet
Affiliation: City University of Hong Kong, HONG KONG
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Abstract: Intrinsic methods in elasticity have been introduced in a landmark series of papers by Wei-Zhang Chien in 1944. During the last two decades, Professor Wojciech Pietraszkiewicz and his group have achieved major advances in their analysis from the mechanical and engineering viewpoints, as well as in their actual numerical implementation. However, it was only in 2005 that their mathematical analysis began to be carried out in earnest, first for three-dimensional elasticity and more recently for elastic shells, by the author and his group. This presentation, which is intended for a general audience, will briefly review and discuss various problems as yet unresolved when this approach is applied to shell structures.

In the classical approach, the main mathematical challenge is to establish that the associated energy has a minimizer. In the linear case, this is achieved through a ”Korn inequality on a surface”, which guarantees the positive-definiteness of the associated energy. In the nonlinear case, the problem remains basically open for Koiter’s model, which is one of the most commonly used nonlinear models in numerical simulations.

In the intrinsic approach, the main challenges lie not only in the mathematical analysis, but in effect in the modeling itself. Since the new unknowns are the change of metric and change of curvature tensor fields (instead of the displacement field in the classical approach), the Gauss and Codazzi-Mainardi compatibility equations conditions (or other equivalent equations) must be satisfied by these new unknowns, in order that they indeed correspond to a displacement of the middle surface of the shell. Another challenge is to adequately express boundary conditions in terms of these new unknowns.

We will briefly review the existence theorems that has been recently obtained in the linear case.
Besides, we will give in particular an explicit form of the compatibility conditions, as well as an explicit "Cesaro-Volterra integral formula on a surface" for reconstructing a displacement field from the knowledge of these new unknowns.

Title: Applications of the Unfolding Method to Some Open Problems in Homogenization

Speaker: Doina Cioranescu
Affiliation: Université Pierre et Marie Curie, FRANCE
Email: cioran@ann.jussieu.fr

Abstract: We will treat some open problems in homogenization by using the periodic unfolding method. The first problem is related to the periodic homogenization for general cases with several micro-scales. The second one concerns perforated domains with “small” holes (i.e., with size going quicker to zero than $\varepsilon$, the periodicity parameter). We then give a homogenization result for a Dirichlet-Neumann problem in a perforated domain with a non homogeneous Neumann boundary condition on the boundary of the holes. Finally, we give a homogenization result for a problem with a Fredholm alternative in a non standard case.

Title: Control and Nonlinearity: Bloch and Navier-Stokes Equations

Speaker: Jean-Michel Coron
Affiliation: Université Pierre et Marie Curie, FRANCE
Email: coron@ann.jussieu.fr

Abstract: In this talk we study the controllability of the Bloch equations (which model an ensemble of non interacting spin $\frac{1}{2}$) and the 3-D Navier-Stokes equations with a force having two vanishing components. In the two cases the linearized control system around the trivial equilibrium is far from being controllable. We show how the return method allows to get controllability results for these two equations. The case of the Bloch equations is a joint work with Karine Beauchard and Pierre Rouchon. The case of the Navier-Stokes equations is a joint work with Pierre Lissy.
Title: Periodic Homogenization for Inner Boundary Conditions with Equi-Valued Surfaces: The Unfolding Approach

Speaker: Aalin Damlamian  
Affiliation: Université Paris XII, FRANCE  
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Abstract: Making use of the Periodic Unfolding Method, we give an elementary proof for the periodic homogenization of the elastic torsion problem of an infinite 3-dimensional rod with multiply-connected cross section as well as for the general electro-conductivity problem in presence of many perfect conductors (arising in resistivity well-logging). Both problems fall into the general setting of equi-valued surfaces with corresponding assigned total fluxes. The unfolding method also gives a general corrector result for these problems.

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Title: Functional Inequalities and the Symmetry Properties of the Extremal Functions

Speaker: Maria J. Esteban  
Affiliation: Université Paris Dauphine, FRANCE  
Email: esteban@ceremade.dauphine.fr

Abstract: In this talk I will present recent work, in collaboration with J. Dolbeault, M. Loss, G. Tarantello and A. Tertikas, about the symmetry properties of extremal functions for (interpolation) functional inequalities playing an important role in the study of long time behavior of evolution diffusion equations. Optimal constants are rarely known, in fact one can write them explicitly only when the extremals enjoy maximal symmetry. This is why the knowledge of the parameters’ regions where symmetry is achieved is of big importance. In the case of symmetry breaking, the underlying phenomena are analyzed.

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Title: On the Numerical Solution of a Nonlinear Wave Equation Associated with the First Painlevé Equation

Speaker: Roland Glowinski  
Affiliation: University of Houston, USA  
Email: roland@math.uh.edu

Abstract: The main goal of this presentation is to address the solution of a nonlinear wave equation associated with one of the celebrated Painlevé equations. The method relies on:  
(i) A time discretization by operator-splitting with control of the time step near singularities.  
(ii) Finite element discretization of the differential operators.  
Actually, the method reduces the solution to the alternate time integrations of a linear wave equation and of the Painlevé ordinary differential equation.  
The method to be discussed in the presentation will be validated by the results of numerical experiments.
Title: Two Topics Related to Nonperiodic Multiscale Problems

Speaker: Claude Le Bris
Affiliation: Ecole des Ponts and INRIA, FRANCE
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Abstract: We present two series of works related to some nonperiodic multiscale problems. The first series consists of joint works with X. Blanc (CEA) and PL. Lions (College de France) and concerns a theory and the derivation of a numerical approach in nonperiodic homogenization, a topic related to materials with defects. The second series of works is a collaboration with A. Lozinski (Toulouse) and F. Legoll (ENPC/INRIA). Multiscale Finite elements methods in the style of Crouzeix Raviart finite elements are derived, tested and theoretically analyzed.

Title: Exact Boundary Synchronization for a Coupled System of Wave Equations

Speaker: Tatsien Li
Affiliation: Fudan University, CHINA
Email: dqli@fudan.edu.cn

Abstract: In this talk, several kinds of exact synchronizations are introduced for a coupled system of wave equations and we show that these kinds of exact synchronizations can be realized by means of boundary controls.

Title: Elliptic Equations With Periodic Coefficients and Homogenization

Speaker: Fang-Hua Lin
Affiliation: New York University, USA
Email: linf@cims.nyu.edu

Abstract: After a brief review of some results concerning elliptic equations with periodic coefficients, I shall discuss two problems that motivate us to the study of various uniform estimates in elliptic homogenizations. In particular, I shall describe some of my recent joint works with C.Kenig and Z.W.Shen on uniform estimates for Neumann problems, the Dirichlet, Neumann and Steklov eigenvalues and (essentially) the optimal convergence rates in homogenization problems.
Title: The Reduced Basis Method: Basic and New Algorithms

Speaker: Yvon Maday
Affiliation: Université Pierre et Marie Curie, FRANCE
Email: maday@ann.jussieu.fr

Abstract: The reduced basis methods (RBM) are discretization methods that allow to get accurate approximations of the solution to partial differential equations when parameters are present in the equations. These methods enter in the frame of reduced model techniques when the (Kolmogorov) dimension of the manifold of all solutions, when the parameter varies, is small. They are not alternative to standard multipurpose discretization methods but are complement approaches to speed up the many resolution of the problem for many parameters that can arise in a control, an optimization of inverse problem is involved. An appropriate selection of the parameters and the computation of approximations of the solutions by the standard method allows to build a new discretization basis that is of small dimension and appropriate to the problem under investigation but not multipurpose. The recent development of the method involves a series of new tools that allows to make these approximations efficient and reliable, in particular with localization and ad hoc adaptations are performed. The purpose of this presentation is to explain the basis of the method, the recent advances and the remaining challenges. Non intrusive approaches will also be presented.

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Title: Mathematical Strategies for Real Time Filtering of Turbulent Dynamical Systems

Speaker: Andrew J. Majda
Affiliation: Courant Institute of Mathematical Sciences, USA
Email: jonjon@cims.nyu.edu

Abstract: An important emerging scientific issue in many practical problems ranging from climate and weather prediction to biological science involves the real time filtering and prediction through partial observations of noisy turbulent signals for complex dynamical systems with many degrees of freedom as well as the statistical accuracy of various strategies to cope with the curse of dimensions. The speaker and his collaborators, Harlim (North Carolina State University), Gershgorin (CIMS Post doc), and Grote (University of Basel) have developed a systematic applied mathematics perspective on all of these issues. One part of these ideas blends classical stability analysis for PDE’s and their finite difference approximations, suitable versions of Kalman filtering, and stochastic models from turbulence theory to deal with the large model errors in realistic systems. Many new mathematical phenomena occur. Another aspect involves the development of test suites of statistically exactly solvable models and new NEKF algorithms for filtering and prediction for slow-fast system, moist convection, and turbulent tracers. Here a stringent suite of test models for filtering and stochastic parameter estimation is developed based on NEKF algorithms in order to systematically correct both multiplicative and additive bias in an imperfect model. As briefly described in the talk, there are both significantly increased filtering and predictive skill through the NEKF stochastic parameter estimation algorithms provided that these are guided by mathematical theory. The recent paper by Majda et al (Discrete and Cont. Dyn. Systems, 2010, Vol. 2, 441-486) as well as a forthcoming introductory graduate text by Majda and Harlim (Cambridge U. Press) provide an overview of this research.
Title: On the problem $-\text{div} A(x)Du = f(x) + H(x, Du) + au$ with $|H(x, Du)| \leq C|Du|^2$ and $a > 0$

Speaker: Francois Murat
Affiliation: Université Pierre et Marie Curie, FRANCE
Email: murat@ann.jussieu.fr

Abstract: The existence of a solution for the problem

$$-\text{div} A(x)Du + a_0(x)u = f(x) + H(x, Du) \text{ in } \Omega, \quad u = 0 \text{ on } \partial \Omega,$$

and a priori bounds for this solution, have been studied in many papers when $A(x)$ is a coercive matrix with $L^\infty(\Omega)$ coefficients, $a_0(x) \in L^\infty(\Omega)$, $f(x) \in L^{N/2}(\Omega)$, and $H(x, Du)$ is a Caratheodory function which satisfies $|H(x, Du)| \leq C|Du|^2$, in the cases where $a_0(x) \geq a_0 > 0$ and where $a_0(x) = 0$. The solution of this problem has to be searched in the space of functions $u \in H^1_0(\Omega)$ which also satisfy $\exp(\gamma|u|) - 1 \in H^1_0(\Omega)$ for a certain $\gamma > 0$. When $a_0(x) = 0$, smallness conditions have to be imposed on the data.

In this talk I will report on recent joint work with Boussad Hamour for the case where $a_0(x)$ has the bad sign, i.e. satisfies $a_0(x) \leq 0$. We obtain the existence of a (small) solution when smallness conditions are imposed on the data.

Title: TBA

Speaker: Jean-Claude Nédélec
Affiliation: Ecole Polytechnique, FRANCE
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Abstract:

Title: Branching Patterns and CellColonies

Speaker: Benoît Perthame
Affiliation: Université Pierre et Marie Curie, FRANCE
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Abstract: The question of understanding the formation of bacterial colonies and their invasion strategy is still under investigation. Several PDE models are known to undergo branching instability. Among those, one of the most famous, inspired by the Gray-Scoot model of chemical reaction, creates dentritic growth; it describes the growth of a cell population under the effect of a nutrient which is locally depleted.

However, a conservative parabolic model that includes the ‘quorum sensing’ limitation has been proposed by Dolak and Schmeiser. The swarmer cells are modeled by a Fokker-Planck type equation à la Keller-Segel. We show that coupled with two fields describing short range attraction and long range repulsion, the model can also undergo branching instabilities.

Several reduced models explain stability and unstability of plateau type traveling wave solutions.
Title: Acceleration Techniques for Stochastic Volatility Models in Finance

Speaker: Olivier Pironneau
Affiliation: Université Pierre et Marie Curie, FRANCE
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Abstract: The equations of mathematical finance can be formulated either as stochastic differential systems (SDEs) or as partial differential equations (PDEs). We shall explore a numerical method where some components are kept as SDEs and other components are replaced by PDEs with the help of Ito calculus.

Mixing Monte-Carlo and PDE methods can substantially speed-up computations because it allows the use of closed form solutions for some of the components. It can also solve problems like unknown boundary conditions for complex stochastic volatility models. Applying the method to stochastic volatility models can speed up the computations by a factor of 40 at least. Furthermore the method extends to American options and to Levy processes. Finally combined with a Longstaff-Schwartz projection it can lead to efficient computations of the complete price surface of early exercise contracts. The convergence of the methods will be established and we shall report numerical performance on multi-dimensional problems. Further accelerations by using POD will be discussed.

Title: A Result of Local Exact Controllability for Viscous Compressible Fluids in 1-d

Speaker: Jean-Pierre Puel
Affiliation: Université de Versailles Saint Quentin, FRANCE
Email: Jean-Pierre.Puel@math.uvsq.fr

Abstract: In the recent years, the controllability problem for viscous incompressible fluids has been extensively studied, trying to answer J.-L. Lions’ questions on the subject, and several results of local exact controllability to trajectories have been obtained.

Up to our knowledge, the corresponding problem for viscous compressible fluids has not been studied except in the last two years in a paper by Amosova and in the article which I will present here written in collaboration by S. Ervedoza, O. Glass, S. Guerrero and J.-P. Puel and which is accepted in Archives for Rational Mechanics and Analysis.

We consider the system modeling a viscous compressible fluid in 1-d on a bounded interval (0, L), namely the equations for the conservation of mass and of momentum. Particular stationary solutions are constant solutions (\( \bar{\rho}, \bar{u} \)). We show that when \( \bar{u} \neq 0 \), if the initial datas \((\rho_0, u_0)\) are in \( H^2(0, L) \) and close enough to \((\bar{\rho}, \bar{u})\), and if the time \( T \) satisfies \( T > L/(\bar{u}) \), there exists a solution \((\rho, u)\) to our system such that \((\rho, u)(0) = (\rho_0, u_0)\) and \((\rho, u)(T) = (\bar{\rho}, \bar{u})\). In this statement, we did not mention any boundary conditions. In fact the boundary conditions are the controls of our problem. In order to obtain this result we use a fixed point argument for a mapping which is not linear but which decouples the controllability for \( u \) and for \( \rho \). In the definition of this mapping,
in order to obtain controllability for $u$ we use Carleman estimates with an adapted weight after an extension of the domain, whereas to obtain controllability for $\rho$, we have to construct $\rho$ using backward and forward characteristics in a rather non standard way. Then, we have to obtain precise estimates in order to use a fixed point argument.

We will present the main steps of this result and some related open problems.

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**Title:** Mathematical Problems for the Equations of the Atmosphere and the Oceans

**Speaker:** Roger Temam

**Affiliation:** Indiana University, USA

**Email:** temam@indiana.edu

**Abstract:** We discuss in this lecture the well-posedness for some linear and nonlinear initial and boundary value problems arising in geosciences.

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**Title:** Beyond Classical Saddle Point Analysis

**Speaker:** Barbara Wohlmuth

**Affiliation:** Technical University of Munich, GERMANY

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**Abstract:** Numerical simulation of coupled problems play an important role in many different fields ranging from industrial and environmental to medical applications. Although of high relevance, the mathematical modeling and the numerical simulation of complex coupled problems remains challenging. This talk adresses two issues: firstly different application areas are illustrated and numerical examples are given. Characteristically for many problems is that the coupling can be realized in terms of balance equations which result in saddle-point like structures. A pair of variables has to guarantee the proper information transfer across the interfaces. In addition to this aspect many complex mathematical models for, e.g., phase transition processes or plasticity involve additional constraints which result in a weak variational inequality setting. In the second part of the talk we review on stable discretization schemes and on optimal a priori estimates for finite element approximations. Here special focus is on the importance of uniform inf-sup stability and the role of surface or volume based Lagrange multipliers.
Title: Control of Viscous Hamilton-Jacobi Equations

Speaker: Enrique Zuazua
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Abstract: We study the problem of null controllability for viscous Hamilton-Jacobi equations in bounded domains of the euclidean space in any space dimension and with controls localized in an arbitrary open non-empty subset of the domain where the equation holds. We prove the null controllability of the system in the sense that, every bounded (and in some cases uniformly continuous) initial datum can be driven to the null state in a sufficiently large time. The proof combines decay properties of the solutions of the uncontrolled system and local null controllability results for small data obtained by means of Carleman inequalities. We also show that there exists a waiting time so that the time of control needs to be large enough, as a function of the norm of the initial data, for the controllability property to hold. We give sharp asymptotic lower and upper bounds on this waiting time both as the size of the data tends to zero and infinity. These results also establish a limit on the growth of nonlinearities that can be controlled uniformly on a time independent of the initial data.

The content of this lecture is based on recent joint work with A. Porretta.