Abstract: We examine the classical problem of ranking a set of players on the basis of a set of pairwise comparisons arising from a sports tournament, with the objective of minimizing the total number of upsets, where an upset occurs if a higher ranked player was actually defeated by a lower ranked player. This problem can be rephrased as the so-called minimum feedback arc set problem on tournaments, which arises in a rich variety of applications and has been a subject of extensive research. We study this NP-hard problem using structure-driven and linear programming approaches.

Let \( T = (V, A) \) be a tournament with a nonnegative integral weight \( w(e) \) on each arc \( e \). A subset \( F \) of arcs is called a feedback arc set if \( T \setminus F \) contains no cycles (directed). A collection \( C \) of cycles (with repetition allowed) is called a cycle packing if each arc \( e \) is used at most \( w(e) \) times by members of \( C \). We call \( T \) cycle Mengerian if, for every nonnegative integral function \( w \) defined on \( A \), the minimum total weight of a feedback arc set is equal to the maximum size of a cycle packing. In this talk, we will discuss the characterization that a tournament is cycle Mengerian if and only if it contains none of four Möbius ladders as a subgraph. (Joint work with Guoli Ding, Wenan Zang, and Qiulan Zhao.)