A SOLUTION TO ERDÖS AND HAJNAL’S ODD CYCLE PROBLEM

Hong Liu
University of Warwick

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Abstract: In 1981, Erdös and Hajnal asked whether the sum of the reciprocals of the odd cycle lengths in a graph with infinite chromatic number is necessarily infinite. Let $C(G)$ be the set of cycle lengths in a graph $G$ and let $C_{\text{odd}}(G)$ be the set of odd numbers in $C(G)$. We prove that, if $G$ has chromatic number $k$, then $\sum_{l \in C_{\text{odd}}(G)} 1/l \geq (1/2 - o_k(1)) \log k$. This solves Erdös and Hajnal’s odd cycle problem, and, furthermore, this bound is asymptotically optimal.

In 1984, Erdös asked whether there is some $d$ such that each graph with chromatic number at least $d$ (or perhaps even only average degree at least $d$) has a cycle whose length is a power of 2. We show that an average degree condition is sufficient for this problem, solving it with methods that apply to a wide range of sequences in addition to the powers of 2.

Finally, we use our methods to show that, for every $k$, there is some $d$ so that every graph with average degree at least $d$ has a subdivision of the complete graph $K_k$ in which each edge is subdivided the same number of times. This confirms a conjecture of Thomassen from 1984. Joint work with Richard Montgomery.