Hypergraphs are mathematical models for many problems in data sciences. In recent decades, the topological properties of hypergraphs have been studied and various kinds of (co)homologies have been constructed. By generalising the usual homology of simplicial complexes, we define the embedded homology of hypergraphs as well as the persistent embedded homology of sequences of hypergraphs. As a generalisation of the Mayer-Vietoris sequence for the homology of simplicial complexes, we give a Mayer-Vietoris sequence for the embedded homology of hypergraphs. We generalize the weighted Laplacians of weighted simplicial complexes to weighted Laplacians of weighted hypergraphs. We study the relations between the weighted Laplacians and the weighted embedded homology of weighted hypergraphs. We generalize the Hodge decompositions of weighted simplicial complexes to Hodge decompositions of weighted hypergraphs. Moreover, as a complement for the Hodge decompositions, we give some results for the nonzero eigenvalues of the weighted Laplacians of weighted hypergraphs.