ON GRAPH NORMS FOR COMPLEX-VALUED FUNCTIONS

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Abstract: For any given graph H, one may define a natural corresponding functional $|.|_H$ for real-valued functions by using homomorphism density. One may also extend this to complex-valued functions, once H is paired with a 2-edge-colouring α to assign conjugates. We say that H is *real-norming* (resp. *complex-norming*) if $|.|_H$ (resp. $|.|_{H,\alpha}$) is a norm on the vector space of real-valued (resp. complex-valued) functions. These generalise the Gowers octahedral norms, a widely used tool in extremal combinatorics to quantify quasirandomness.

We unify these two seemingly different notions of graph norms in real- and complex-valued settings. Namely, we prove that H is complex-norming if and only if it is real-norming and simply call the property *norming*. Our proof does not explicitly construct a suitable 2-edge-colouring α but obtains its existence and uniqueness, which may be of independent interest.

As an application, we give various example graphs that are not norming. In particular, we show that hypercubes are not norming, which resolves the last outstanding problem posed in Hatamis pioneering work on graph norms.

Joint work with Sasha Sidorenko.