Asymptotic properties of solutions to equations of fluid dynamics

Eduard Feireisl *

Institute of Mathematics of the Academy of Sciences of the Czech Republic;
Žitná 25, CZ-115 67 Praha 1, Czech Republic
Institute of Mathematics, Technische Universität Berlin,
Straße des 17. Juni 136, 10623 Berlin, Germany
feireisl@math.cas.cz

Abstract

The goal of this lecture series is to give an overview of some recent results and new directions in continuum fluid dynamics. After a brief introduction of the basic system of balance laws, we focus on two iconic examples: The Navier–Stokes(–Fourier) system describing the motion of a general compressible viscous and heat conducting fluid, and the Euler system describing the motion of a perfect fluid, where dissipation effects are neglected.

Keywords: Euler system, Navier–Stokes system, Lax equivalence principle, Cesàro average, generalized solutions

1 Lecture 1: Fluid equations in the framework of continuum mechanics

Field equations in fluid mechanics: Conservation/balance law, strong and weak formulation. Constitutive equations. The Navier–Stokes and Euler systems. Weak vs. strong solutions, blow up of strong solutions of the Euler system. Almost explicit solutions to the stationary problem. Well posedness theory – the state-of-the-art, compensated compactness vs. convex integration.

Relevant reference material: Abbatiello and EF [1], De Lellis and Székelyhidi [8], EF and Novotný [15], Gallavotti [17], Tartar [19]

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2 Lecture 2: Stability and numerical analysis: Lax equivalence principle

Consistent approximation. A priori estimates and stability. Asymptotic limit of consistent approximations. Generalized solutions – weak, measure–valued and beyond. Lax equivalence principle in the context of nonlinear problems. Convergence of consistent approximations.

Relevant reference material: DiPerna and Majda [9], DiPerna [10], EF and Hofmanová [12], EF, Karper, Pokorný [13], EF and Lukáčová [14]

3 Lecture 3: Method of averaging

Examples of averaging: Ergodic theory, Banach Sachs/Komlos theorems, Young measures. (S)-convergence and its applications in the analysis of weakly converging consistent approximations. Global and stationary solutions.

Relevant reference material: Balder [2], EF [11], Komlós [18]

4 Lecture 4: Asymptotic behaviour of solutions for large time

Long time behavior of solutions of the Euler system. Semigroup selection and admissibility criteria. Statistical solutions. Long time behavior of the Navier–Stokes system, asymptotic compactness and attractors.

Relevant reference material: Breit, EF, Hofmanová [4], Cardona and Kapitanskii [5], EF and Pražák [16]

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