On product sets of arithmetic progressions

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Link: https://zoom.us/j/89190957929

Abstract:

We prove that the size of the product set of any finite arithmetic progression $\mathcal{A} \subset \mathbb{Z}$ satisfies

$$|\mathcal{A} \cdot \mathcal{A}| \geq \frac{|\mathcal{A}|^2}{(\log |\mathcal{A}|)^{2\theta + o(1)}},$$

where $2\theta = 1 - (1 + \log\log 2)/(\log 2)$ is the constant appearing in the celebrated Erdős multiplication table problem. This confirms a conjecture of Elekes and Ruzsa from about two decades ago.

If instead $\mathcal{A}$ is relaxed to be a subset of a finite arithmetic progression in integers with positive constant density, we prove that

$$|\mathcal{A} \cdot \mathcal{A}| \geq \frac{|\mathcal{A}|^2}{(\log |\mathcal{A}|)^{2\log 2 - 1 + o(1)}}.$$

This solves the typical case of another conjecture of Elekes and Ruzsa on the size of the product set of a set $\mathcal{A}$ whose sum set is of size $O(|\mathcal{A}|)$. Joint work with Max Wenqiang Xu.