复旦大学数学科学学院

数学综合报告会

报告题目：A fully discrete finite difference scheme for the Flory-Huggins-Cahn-Hilliard equation with dynamical boundary conditions

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地点：Zoom网址：https://umassd.zoom.us/j/6339263022，密码：123456

报告摘要：

A fully discrete finite difference numerical scheme is proposed and analyzed for the Cahn-Hilliard equation with a logarithmic Flory Huggins energy potential, combined with the dynamic boundary condition. Such a boundary condition couples the interior evolution of the gradient flow with the boundary evolution, and a dissipation law becomes available for the total energy, including the bulk energy and surface energy. The centered finite difference spatial approximation is taken. In the temporal discretization, we treat the nonlinear logarithmic terms and the surface diffusion term implicitly, and update the linear expansive term explicitly. Such a convex splitting approach is applied at both the interior dynamics and the boundary evolution. In turn, a careful calculation reveals that, the implicit part of the numerical system corresponds to a minimization of strictly convex discrete energy functional. In particular, the coefficient for the singular logarithmic terms becomes $(1 + 2 h^{-1})$ on the boundary points, in comparison with the regular coefficient, given by 1, at interior grid points. This subtle fact leads to the well-posed feature of the numerical system. A theoretical justification of the unique solvability and positivity-preserving property of this numerical algorithm is provided, so that the phase variable is always between -1 and 1 at a point-wise level. This analysis is based on the following fact: the singular nature of the logarithmic term around the values of -1 and 1 prevents the numerical solution reaching these singular values, so that the numerical scheme is always well-defined as long as the numerical solution stays bounded at the previous time step. In addition, an unconditional energy stability of the numerical scheme is derived, without any restriction for the time step size. The optimal rate convergence analysis is presented as well.

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