Microlocal Analysis and Applications

Shanghai Centre for Mathematical Sciences, June 2019

Introduction

Microlocal analysis originated in the 1950s from the use of Fourier transform techniques in the study of variable-coefficient PDEs; its intellectual roots lie in geometric optics and the WKB approximation. The field took on a coherent identity starting in the 1960s with the development of pseudodifferential and, later, Fourier integral operators as fundamental tools. Since then, microlocal analysis has seen a remarkable variety of applications across pure and applied mathematics and physics. Just within the last several years, the field has witnessed both striking breakthroughs on known microlocal problems and spectacular new results in areas where microlocal analysis had not previously been viewed as a natural tool. Among the exciting recent developments that the conference will focus on are:

- The proof of the nonlinear stability of Kerr-de Sitter families due to Hintz–Vasy
- The work of Faure–Sjöstrand on Anosov flows and Dyatlov–Zworski's subsequent proof of the meromorphic continuation of the Ruelle zeta function
- Boundary rigidity on general Riemannian manifolds with convex boundaries due to Stefanov–Uhlmann–Vasy
- Dyatlov–Jin's proof that quantum limit measures on hyperbolic surfaces have compact support, and related applications of the fractal uncertainty principle.

This conference, the first ever event in China with an emphasis on microlocal analysis, will feature talks by many leaders in the field. It should provide a good oppportunity for students and postdocs entering the field or interested in learning microlocal methods to learn the state of the art.

Topics

- Microlocal analysis and semiclassical analysis,

- Spectral and scattering theory,
- Microlocal methods in inverse problems,
- Microlocal methods in general relativity.

Organizers

- Chen Xi (Cambridge)
- Du Kai (Fudan)
- Colin Guillarmou (Paris-Sud)
- Huang Genggeng (Fudan)
- Jared Wunsch (Northwestern)

Support

- China National Natural Science Foundation
- Shanghai Center for Mathematical Sciences

Short courses

Schedule

	9:00-10:00	10:30-11:30	14:00-15:00	15:30-16:30
14 June (Fri)	Baskin	Wunsch	Baskin	Salo
15 June (Sat)	Wunsch	Jin	Salo	Jin

Venue

Room 1801, Guanghua East Tower, **Handan Campus**, Fudan University.

Courses

- **Dean Baskin** (TAMU) : *Introduction to semiclassical microlocal analysis*

  Semiclassical analysis is motivated in large part by the quantum-classical correspondence in quantum mechanics. It provides a framework to study solutions of differential equations containing a small positive parameter h. In the first lecture, I will give an axiomatic description of the semiclassical pseudodifferential calculus and show how it can be used to prove that elliptic operators are nearly invertible. In the second lecture, I will introduce semiclassical defect measures, which provide a description of how sequences of functions concentrate in phase space. If the sequence satisfies a differential equation (such as a sequence of eigenfunctions for the Laplacian), then the limiting measure is invariant under an associated flow.

- **Jin Long** (Tsinghua) : *Introduction to quantum ergodicity*

  In this minicourse, we discuss the phenomenon of quantum ergodicity. Roughly speaking, quantum ergodicity states that if the underlying dynamics of the classical system is ergodic, then the eigenstates in the corresponding quantum system

Figure 1: Guanghua Twin Towers

tend to uniform distribution in the high energy limit. Such results were first stated by Shnirelman, and proved by Colin de Verdiére and Zelditch. We will present a version for semiclassical Schrödinger operators on a compact manifold.

- **Mikko Salo** (Jyväskylä) : *Applications of microlocal analysis in inverse problems*

  Typical inverse problems include recovering an unknown function from its integrals over curves (X-ray transform), or recovering an unknown coefficient in an elliptic or hyperbolic PDE from boundary measurements (Caldern or Gel'fand problem). The boundary measurement operators are often pseudodifferential or Fourier integral operators. In this minicourse we will explain how microlocal analysis can be used to solve inverse problems of this type.

- **Jared Wunsch** (Northwestern) : *Applications of microlocal analysis to evolution equations*

  Semiclassical analysis of the resolvent of the Laplacian (a.k.a. uniform analysis as the spectral parameter tends to infinity, near the real axis) is a powerful tool for studying the behavior of solutions to evolution equations. We will discuss how the methods of semiclassical analysis yield resolvent estimates, and then see how to use those resolvent estimates to study the dispersive behavior of wave and Schrödinger equations.

Conference Talks

Schedule

	17 Jun (Mon)	18 Jun (Tue)	19 Jun (Wed)	20 Jun (Thu)	21 Jun (Fri)
09:00-09:50	Opening	Borthwick	Faure	Schrohe	Bouclet
09:50-10:40	Uhlmann	Salo	Tzou	Gomes	Fermanian
10:40-11:10	Tea	Tea	Tea	Tea	Tea
11:10-12:00	Datchev	Baskin	Christianson	Wang Z.Q.	Zelditch
14:00-14:50	Dyatlov	Tacy		Hintz	
14:50-15:20	Tea	Tea		Tea	
15:20-16:10	Zhu	Canzani		Wang F.	
16:10-17:00		Galkowski		Jin	
18:30-20:30	Dinner By invitation				

Venue

The conference will take place in Gu Lecture Hall, SCMS Building, **Jiangwan Campus**, Fudan University. The conference dinner will take place in Danyuan Restaurant. The following coach services are provided for invited participants during the main conference.

- From Baolong to SCMS: 8:30am, 17-21 June (Mon-Fri).
- From SCMS to Baolong: 17:30pm, 18 June (Tue), 20 June (Thu).
- From SCMS to Baolong: 13:00pm, 19 June (Wed), 21 June (Fri).
- From SCMS to Danyuan: 17:00pm, 17 June (Mon)
- From Danyuan to Baolong: 20:30pm, 17 June (Mon)

Figure 2: The SCMS Building

Title/Abstract

- **Dean Baskin** (TAMU) : *Diffraction for the Dirac equation by Coulomb-like potentials*

  The Dirac equation describes the relativistic evolution of electrons and positrons. We consider the (time-dependent!) Dirac equation in three dimensions coupled to a potential with Coulomb-type singularities. We prove a propagation of singularities result for this equation and show that singularities are typically diffracted by the singularities of the potential. We finally compute the symbol of the diffracted wave. This talk is based on joint work with Oran Gannot and Jared Wunsch.

- **David Borthwick** (Emory) : *Dispersive estimates and Schrödinger solitons on hyperbolic manifolds*

  We some recent dispersive estimates for hyperbolic and asymptotically hyperbolic manifolds, and then discuss applications to the problem of existence and stability for solitons of the focusing nonlinear Schrödinger equation. (Based on joint work with Roland Donninger, Enno Lenzmann, and Jeremy Marzuola.)

- **Jean-Marc Bouclet** (Toulouse) : *Propagation of coherent states on Riemannian manifolds*

  Coherent states (or Gaussian wave packets) have proved to be very useful in the description of the propagator of Schrödinger operators on the Euclidean space (as e.g. in the works of Combescure-Robert). In this talk we shall describe an

(almost intrinsinc) analoguous approach on Riemannian manifolds. This allows in particular to capture fairly directly the influence of negative curvature on the evolution over the Ehrenfest time scale.

- **Yaiza Canzani** (UNC) : *Understanding the growth of Laplace eigenfunctions*

  In this talk we will discuss a new geodesic beam approach to understanding eigenfunction concentration. We characterize the features that cause an eigenfunction to saturate the standard supremum bounds in terms of the distribution of L^2 mass along geodesic tubes emanating from a point. We also show that the phenomena behind extreme supremum norm growth is identical to that underlying extreme growth of eigenfunctions when averaged along submanifolds. Using the description of concentration, we obtain quantitative improvements on the known bounds in a wide variety of settings.

- **Hans Christianson** (UNC) : *Easy theorems for some simple boundary value problems in Euclidean domains*

  In this talk I will describe a number of interesting results about eigenfunctions on simple Euclidean domains. This includes distribution of mass on the interior and for boundary values, as well as applications to boundary observability for the wave equation. The proofs are short and easy, using mainly integrations by parts. Parts of this work are with Katrina Lu, Evan Stafford, John Toth, and Jin Xi.

- **Kiril Datchev** (Purdue) : *Semiclassical resolvent estimates for rough potentials*

  Semiclassical resolvent estimates are important for their applications to resonance free regions, wave decay, and other problems in scattering theory. Typically the norm of the resolvent may be as bad as exponentially large near a trapped set, but must be much smaller far away from it. This talk will survey our understanding of these phenomena in the setting of Euclidean potential scattering, including remaining open problems. Based on joint work with Maarten de Hoop, Long Jin, and Jacob Shapiro.

- **Semyon Dyatlov** (Berkeley) : *Control of eigenfunctions on negatively curved surfaces*

  Given an L^2-normalized eigenfunction with eigenvalue λ^2 on a compact Riemannian manifold (M, g) and a nonempty open set $\Omega \subset M$, what lower bound can we prove on the L^2-mass of the eigenfunction on Ω? The unique continuation

principle gives a bound for any Ω which is exponentially small as $\lambda \to \infty$. On the other hand, microlocal analysis gives a λ-independent lower bound if Ω is large enough, i.e. it satisfies the geometric control condition.

This talk presents a λ-independent lower bound for any set Ω in the case when M is a negatively curved surface, or more generally a surface with Anosov geodesic flow. The proof uses microlocal analysis, the chaotic behavior of the geodesic flow, and a new ingredient from harmonic analysis called the Fractal Uncertainty Principle. Applications include control for Schrödinger equation and exponential decay of damped waves. Joint work with Jean Bourgain, Long Jin, and Stéphane Nonnenmacher.

- **Frédéric Faure** (Grenoble) : *Emergence of the quantum wave equation in classical deterministic hyperbolic dynamics*

  In the 80's, D. Ruelle, D. Bowen and others have introduced probabilistic and spectral methods in order to study deterministic chaos ("Ruelle resonances"). For a geodesic flow on a strictly negative curvature Riemannian manifold, following this approach and use of microlocal analysis, one obtains that long time fluctuations of classical probabilities are described by an effective quantum wave equation. This may be surprising because there is no added quantization procedure. We will discuss consequences for the zeros of dynamical zeta functions. This shows that the problematics of classical chaos and quantum chaos are closely related. Joint work with Masato Tsujii.

- **Clotilde Fermanian** (UPEC) : *Semi-classical analysis on H-type groups*

  We present in this talk recent results obtained in collaboration with Vronique Fischer (University of Bath, UK) aiming at developing a semi-classical approach in sub-Laplacian geometry. In the context of H-type groups, we describe how to construct a semi-classical pseudodifferential calculus compatible with the Lie group structure and we discuss the associated notion of semi-classical measures, together with some of their properties.

- **Jeffrey Galkowski** (Northeastern) *Geodesic Beams in Eigenfunction Analysis*

  This talk is a continuation of 'Understanding the growth of Laplace eigenfunctions'. We explain the method of geodesic beams in detail and review the development of these techniques in the setting of defect measures. We then describe the tools and give example applications in concrete geometric settings.

- **Sean Gomes** (Northwestern) *Semiclassical scarring in KAM Hamiltonian systems*

  In this talk we will discuss the phenomenon of semiclassical scarring in KAM Hamiltonian systems in dimension 2. The persistence of a large measure family of flow-invariant Lagrangian tori in the classical dynamics is reflected on the quantum side by the scarring of quantum limits of eigenfunctions on almost all of these tori for a generic family of perturbations.

  A key ingredient in the proof are the quasimodes associated to such tori, constructed by Colin de Verdiére and subsequently Popov in the Gevrey regularity setting.

  This talk is based on joint work with Andrew Hassell.

- **Peter Hintz** (MIT) *Linear stability of slowly rotating Kerr spacetimes*

  I will describe joint work with Dietrich Häfner and András Vasy in which we study the asymptotic behavior of linearized gravitational perturbations of Schwarzschild or slowly rotating Kerr black hole spacetimes. We show that solutions of the linearized Einstein equation decay at an inverse polynomial rate to a stationary solution (given by an infinitesimal variation of the mass and angular momentum of the black hole), plus a pure gauge term. Our proof uses a detailed description of the resolvent of an associated wave equation on symmetric 2-tensors near zero energy.

- **Jin Long** (Tsinghua) *Resonances for open quantum maps*

  Open quantum maps are useful models in the study of scattering resonances, especially for open quantum chaotic systems. In this talk, we discuss a special family of open quantum maps known as open quantum baker's maps. They are quantizations of the open baker's maps on the torus and are given by a family of subunitary matrices. We are interested in the distribution of eigenvalues of these subunitary matrices, which are analogues of scattering resonances in this setting. In particular, we show that there exists a spectral gap which improves both the trivial gap and the pressure gap. We also prove a fractal Weyl upper bound for the number of eigenvalues in annuli. This is based on joint work with Semyon Dyatlov.

- **Mikko Salo** (Jyväskylä) : *The fixed angle inverse scattering problem*

  We consider the fixed angle inverse scattering problem for potentials in Euclidean space. The main result shows that a compactly supported potential is

uniquely determined by its scattering amplitude for two opposite fixed angles. We also show that potentials having a generalized reflection symmetry are uniquely determined by their fixed angle scattering data. The proofs are based on reducing the inverse scattering problem to a unique continuation type problem for the wave equation in the spirit of the Bukhgeim-Klibanov method, and on using suitable Carleman estimates.

This is a joint work with Rakesh (Delaware).

- **Elmar Schrohe** (Hannover) : *Bounded H_∞-calculus for Realizations of Elliptic Boundary Problems on Manifolds with Conical Singularities*

  We consider elliptic differential boundary value problems on manifolds with conical singularities. In order to simplify the analysis, we blow up the singular points and study conically degenerate operators on the resulting object.

  We first present a simplified way of determining the closed realizations in weighted Mellin-Sobolev spaces. Then we show that a stronger condition, namely the parameter-ellipticity of a realization, guarantees the existence of a bounded H_∞-calculus.

  We illustrate this on the example of the Dirichlet and Neumann Laplacian. As an application we establish the existence of short time solutions for the porous medium equation with Neumann boundary conditions on manifolds with (possibly warped) cones for positive initial data.

  (Joint work with Nikolaos Roidos (Patras, Greece) and Jörg Seiler (Turin, Italy))

- **Leo Tzou** (Sydney) : *Applications of semiclassical resolvents to tomography problems*

  A classical result of Jerison-Kenig showed that the optimal assumption for unique continuation properties for elliptic PDE. In this talk we will explore its connection to image reconstruction with impedance tomography.

  We will develop an analogous theory in the context of partial data inverse problems to obtain the same sharp regularity assumption as Jerison-Kenig. The method we use involves explicit microlocal construction of the Dirichlet Green's function which on its own may be of interest for partial data image reconstruction.

- **Gunther Uhlmann** (Seattle and Hong Kong) : *Inverse Problems in Wave Propagation*

We describe the solution of several inverse problems arising in wave propagation using the methods of microlocal analysis.

- **Wang Fang** (SJTU) : *Eigenvalue comparison theorems for Poincare-Einstein manifolds*

  In this talk, I will give several eigenvalue comparison theorems associated to Poincare-Einstein manifolds, as well as characterisation of equalities, which leads to some rigidity theorems.

- **Wang Zuoqin** (USTC) : *Equivariant Eigenvalues for Manifolds with Large Symmetry*

  Consider a Riemannian manifold M whose Riemannian metric is invariant under the action of a compact Lie group G. Then the Laplacian spectra decomposes naturally according to the irreducible representations of G. I will explain the role of symplectic geometry in studying these equivariant eigenvalues. In particular, I will show how to apply symplectic techniques to study inverse spectral problems on manifolds with large symmetry, especially on toric manifolds. This is based on joint works with V. Guillemin, A. Uribe and with Y. Qin.

- **Steven Zelditch** (Northwestern) : *Observability estimate from a hyper surface*

  The basic question is: find necessary and sufficient conditions that the restriction of a sequence of eigenfunctions to a hypersurface H tends to zero in $L^2(H)$. In particular, if H is a curve on a hyperbolic surface, what does the Dyatlov-Jin observability estimate imply about the sequence?

- **Zhu Xuwen** (Berkeley) : *Spherical conical metrics*

  The problem of finding and classifying constant curvature metrics with conical singularities has a long history bringing together several different areas of mathematics. This talk will focus on the particularly difficult spherical case where many new phenomena appear. When some of the cone angles are bigger than 2π, uniqueness fails and existence is not guaranteed; smooth deformation is not always possible and the moduli space is expected to have singular strata. I will give a survey of several recent results regarding this singular uniformization problem, connecting microlocal techniques with complex analysis and synthetic geometry. Based on joint works with Rafe Mazzeo and Bin Xu.

Practical Information

Location

The summer school (14-15 June) and the main conference (17-21 June) will take place in Handan Campus (220 Handan Rd) and Jiangwan Campus (2005 Songhu Rd) respectively.

- Handan (Campus) is within walking distance of 'Jiangwan Stadium' station or 'WuJiaoChang' station of Metro Line 10. The School of Mathematical Sciences (http://math.fudan.edu.cn/) is in the Guanghua East Tower in Handan.

- Jiangwan (Campus) is close to the 'XinJiangWanCheng' station of Metro Line 10 (the terminal of Line 10 for the time being). Both the Shanghai Centre for Mathematical Sciences (http://www.scms.fudan.edu.cn/) and the SCMS Guest House are located in Jiangwan.

Transportation

The Metro system in Shanghai is highly developed and covers most of the urban regions. It is punctual but might get crowded during rush hours. On the other hand, the journey from Pudong airport to the city is very time consuming.

Taxi is more expensive and faster. But sometimes it is hard to find one. Please have the address handy and show it to the driver.

In addition, there is a super fast maglev line from Pudong airport to 'LongYang Road' station of Metro Line 2 (in the eastern part of the city). Please remember to have your boarding pass or your itinerary with you for the concession price.

The airport shuttle bus for Pudong Airport is cheaper than the cab and faster than the Metro. But it might be tricky for non-Chinese speakers.

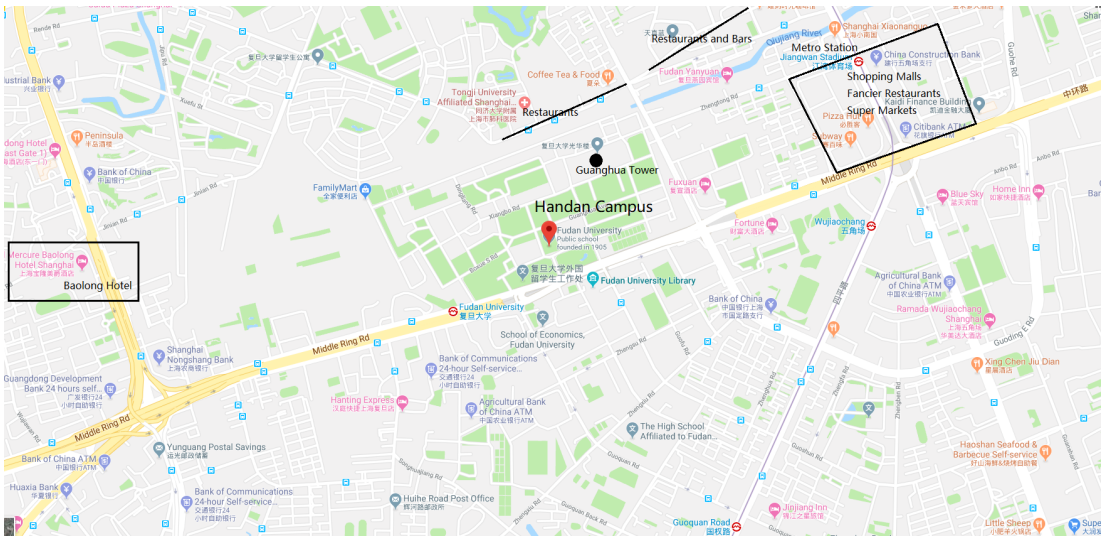

Figure 3: Handan Campus

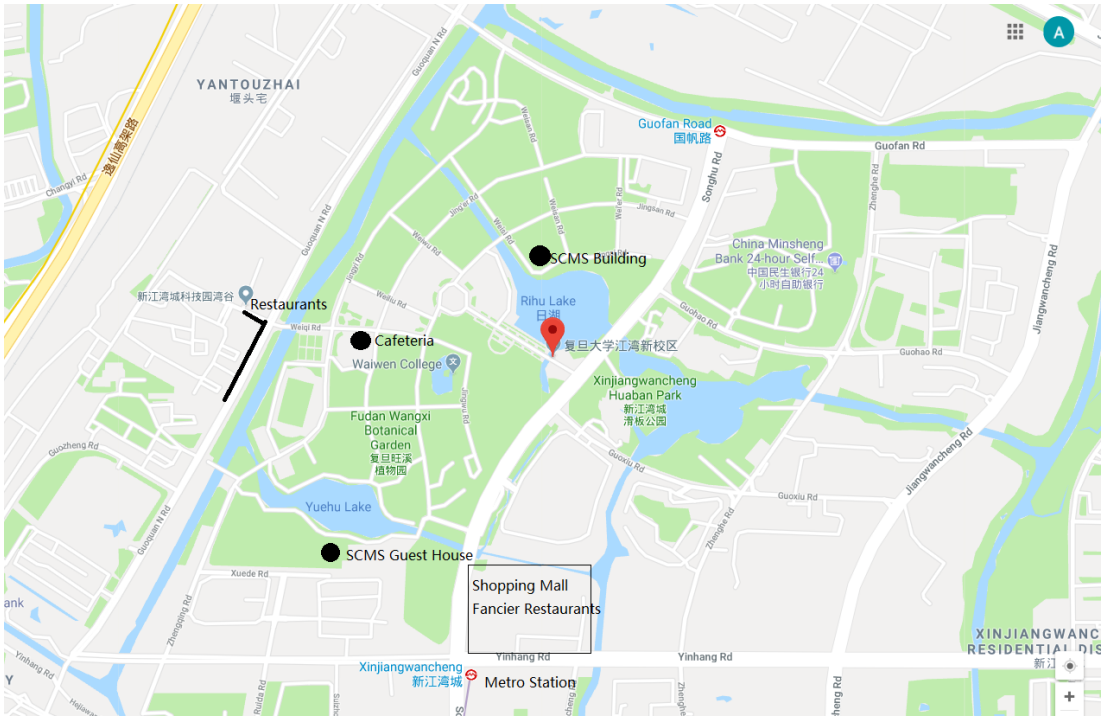

Figure 4: Jiangwan Campus

Routes

- From Pudong Airport to Baolong Hotel:
  - By Metro: Take Line 2 at the airport, transfer to Line 4 (anti-clockwise) at Century Avenue, transfer to Line 3 (northbound) at 'Baoshan Road', take off at 'Jiangwan Town'. Please note Baolong hotel is 1.2 km away from the station.
  - By Cab: It costs 150-200 yuan from Pudong Airport to Baolong Hotel.
  - By Maglev: Take the maglev line to 'Longyang Road'. Continue to take a cab or Metro Line 2 afterwards.
  - Shuttle Bus: Take Shuttle Bus Line 4 to Hongkou Stadium, take Metro Line 3 (northbound), take off at 'Jiangwan Town'.
- From Pudong Airport to Jiangwan Campus / Handan Campus:
  - By Metro: Take Line 2 at the airport, transfer to Line 10 (northbound) at 'Nanjing East Road', take off at 'Jiangwan Stadium' / 'XinJiangWanCheng' for Handan Campus / Jiangwan Campus.
  - By Cab: It costs 150-200 yuan from Pudong Airport to Baolong Hotel
  - By Maglev: Take the maglev line to 'Longyang Road'. Continue to take a cab or Metro Line 2 afterwards.
  - Shuttle Bus: Take Shuttle Bus Line 4 and take off at 'WuJiaoChang'. Handan Campus is about 2 min walk from the stop. If you go to Jiangwan Campus, continue to take Metro Line 10 (northbound) to 'XinJiangWanCheng'.
- From Hongqiao Airport / Rail Station to Jiangwan Campus / Handan Campus :
  - By Metro: Take Line 10 at the airport / rail station and take off at 'Jiangwan Stadium' for Handan Campus or take off at 'XinJiangWanCheng' for Jiangwan Campus.
  - By Cab: It costs 100-150 yuan from Hongqiao Airport / Rail Station to Jiangwan Campus / Handan Campus.
- From Hongqiao Airport / Rail Station to Baolong Hotel / Handan Campus :

- By Metro: Take Line 10 at the airport / rail station, transfer to Line 3 (northbound) at 'Hongqiao Road', take off at 'Jiangwan town'. Please note Baolong hotel is 1.2 km away from the station.
- By Cab: It costs 100-150 yuan from Hongqiao Airport / Rail Station to Jiangwan Campus / Handan Campus.

University Cafeteria

Invited speakers will be given lunch tickets to have lunch in the university cafeteria in Jiangwan during the main conference.

The university cafeterias, being subsidized, are not really open to public. That being said, other participants, in principle, need a meal card to make the payment. Visitors to the SCMS can borrow university's visitor cards from the SCMS admin office (a refundable deposit of 200 yuan is required). The visitor cards can be topped up in the cafeterias during lunch hours on Tue and Fri. Since the number of available visitor cards is very limited, a visitor card might have to be shared by a few visitors. Please note that the balance in the card is NOT refundable.

In addition, the third floor of the cafeteria in Jiangwan accepts the payment via WeChat or AliPay, whilst participants staying at the SCMS guest house can purchase breakfast tickets at the guest house.

Alternatively, there are considerable places in the surrounding streets. See the Figures.

Minor Remarks

- The tap water in China is NOT potable unless there is a sign saying it is drinking water.
- It is convenient to have a public transportation card to use Metro or Bus in Shanghai. But it is better to consult the staff about the refunding policy before you buy the card.
- The bus service in Shanghai is very cheap (2 yuan for a single ride). But no change is available in the bus. Please have exact amount of coins or a public transportation card.
- Do NOT expect the taxi drivers speak English. It is important to have with you the address or name of your destination in Chinese.

- A valid passport or national ID card is always required for check-in in a hotel.

Emergency Contacts

- Police: 110
- Fire services: 119
- Ambulance: 999