Density of C_{-4}-critical signed graphs

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Abstract:

A signed graph (G, σ) is a graph G together with a signature $\sigma : E(G) \to \{+, -\}$. A homomorphism of a signed graph (G, σ) to another signed graph (H, π) is a mapping from $V(G)$ to $V(H)$ such that the adjacency and the signs of closed walks are preserved. Given a signed graph (G, σ), let $g_{ij}(G, \sigma)$ $(ij \in \mathbb{Z}_2^2)$ denote the length of a shortest non-trivial closed walk whose parity of the number of negative edges is equal to i modulo 2 and parity of the length is equal to j modulo 2. We observe that if (G, σ) admits a homomorphism to (H, π), then $g_{ij}(G, \sigma) \geq g_{ij}(H, \pi)$ for each $ij \in \mathbb{Z}_2^2$. A signed graph (G, σ) is (H, π)-critical if it satisfies that $g_{ij}(G, \sigma) \geq g_{ij}(H, \pi)$, and it admits no homomorphism to (H, π) but each of its proper subgraphs does.

By a signed indicator construction, we first show that the k-coloring problem of graphs is captured by the C_{-k}-coloring problem of signed graphs. Then we prove that, except for one particular signed graph on 7 vertices and 9 edges, any C_{-4}-critical signed graph on n vertices must have at least $\lceil \frac{4n}{3} \rceil$ edges. Moreover, for each value of $n \geq 9$, there exists a C_{-4}-critical signed graph on n vertices with either $\lceil \frac{4n}{3} \rceil$ or $\lceil \frac{4n}{3} \rceil + 1$ many edges. As an application to planar graphs, we conclude that every signed bipartite planar graphs of negative-girth at least 8 admits a homomorphism to C_{-4} and, furthermore, this bound is the best possible. This fits well into a larger framework of the study of analog of Jaeger-Zhang conjecture.

This is joint work with Reza Naserasr and Lan Anh Pham.