A SOLUTION TO ERDÖS AND HAJNAL'S ODD CYCLE PROBLEM

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Abstract: In 1981, Erdös and Hajnal asked whether the sum of the reciprocals of the odd cycle lengths in a graph with infinite chromatic number is necessarily infinite. Let $C(G)$ be the set of cycle lengths in a graph G and let $C_{odd}(G)$ be the set of odd numbers in $C(G)$. We prove that, if G has chromatic number k, then $\sum_{l \in C_{odd}(G)} 1/l \geq (1/2 - o_k(1)) \log k$. This solves Erdös and Hajnal's odd cycle problem, and, furthermore, this bound is asymptotically optimal.

In 1984, Erdös asked whether there is some d such that each graph with chromatic number at least d (or perhaps even only average degree at least d) has a cycle whose length is a power of 2. We show that an average degree condition is sufficient for this problem, solving it with methods that apply to a wide range of sequences in addition to the powers of 2.

Finally, we use our methods to show that, for every k, there is some d so that every graph with average degree at least d has a subdivision of the complete graph K_k in which each edge is subdivided the same number of times. This confirms a conjecture of Thomassen from 1984. Joint work with Richard Montgomery.