


复旦大学数学科学学院
数学综合报告会

报告题目：Aspects of Existence and Regularity Theory in the Calculus of Variations

报告人：Professor Jan Kristensen（University of Oxford, UK）

时间：2021-06-21 星期一 16:30 – 17:30; 2021-06-23 星期三 16:30 – 17:30; 2021-06-2 8 星期一 16:30-17:30; 2020-06-30 星期三 16:30-17:30

地点：ZOOM Meeting ID: 876 1464 0705，Passcode: 753770

报告摘要：

It has been known for some time that questions about the existence and regularity of minimizers for a variational integral on a Dirichlet class are intimately connected to quasiconvexity properties of the corresponding integrand. Quasiconvexity was introduced by Morrey in 1952 in connection with his work on the semicontinuity of variational integrals. Under suitable technical assumptions, it is closely related to the existence of a minimizer in a given Dirichlet class. In turn, quasiconvexity is defined by requiring the minimality of linear mappings over their Dirichlet class. As such it is only marginally more transparent than the problem it was intended to solve. Indeed, it is often very hard to prove or disprove that a given integrand is quasiconvex, and well-known examples of integrands are recorded in the literature where their quasiconvexity (at the origin) would have many interesting ramifications. Strict quasiconvexity, suitably quantified, of the integrand at \textit{some} point, is essentially equivalent to coercivity of the variational integral on Dirichlet classes. On the other hand, for integrands satisfying the condition for coercivity \textit{everywhere} (so strongly quasiconvex integrands), Evans derived in 1986 Caccioppoli inequalities of the second kind for the minimizers of the corresponding variational integrals. As a consequence, he established ε-regularity results, and hence partial regularity of such minimizers. While the above results are all subject to the condition of quasiconvexity in some form, and therefore can appear superficial, they do gain some traction when it is recalled that quasiconvexity can be framed between rank-one convexity (a necessary condition) and polyconvexity (a sufficient condition).

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