

五号问题

设 m, n 均为正整数, $n > m$.

$$\begin{aligned} f(y) &= m^2(n-m)(y^{m+3n} - 1) \\ &\quad - m^2(n+m)(y^{3n} - y^m) \\ &\quad - n^2(n-m)(y^{2m+2n} - y^{n-m}) \\ &\quad - (n+m)(n^2 - 4nm + m^2)(y^{2n+m} - y^n) \\ &\quad - n^2(n+m)(y^{n+2m} - y^{2n-m}) \\ &\quad + (n-m)(n^2 + 4nm + m^2)(y^{2n} - y^{n+m}). \end{aligned}$$

记

$$f(x+1) = a_N x^N + a_{N-1} x^{N-1} + \dots + a_1 x + a_0.$$

证明或否定: 存在 $k = k(m, n)$, 使得

$$\begin{cases} a_j \geq 0, & \forall j \geq k, \\ a_j \leq 0, & \forall j < k. \end{cases}$$