Covering cubes by hyperplanes

Hao Huang (Emory University)

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Abstract: Note that the vertices of the n-dimensional cube $\{0,1\}^n$ can be covered by two affine hyperplanes $x_1 = 1$ and $x_1 = 0$. However if we leave one vertex uncovered, then suddenly at least n affine hyperplanes are needed. This was a classical result of Alon and Füredi, followed from the Combinatorial Nullstellensatz.

In this talk, we consider the following natural generalization of the Alon-Füredi theorem: what is the minimum number of affine hyperplanes such that the vertices in $\{0,1\}^n \setminus \{\vec{0}\}$ are covered at least k times, and $\vec{0}$ is uncovered? We answer the problem for $k \leq 3$ and show that a minimum of $n+3$ affine hyperplanes is needed for $k = 3$, using a punctured version of the Combinatorial Nullstellensatz. We also develop an analogue of the Lubell-Yamamoto-Meshalkin inequality for subset sums, and solve the problem asymptotically for fixed n and $k \to \infty$, and pose a conjecture for fixed k and large n.

Joint work with Alexander Clifton (Emory University).

About the speaker: Hao Huang is currently a tenure-track Assistant Professor in the Math & CS Department at Emory University. He received the Ph.D. degree in Jun, 2012, from Department of Mathematics, UCLA, under the supervision of Professor Benny Sudakov. He recently proved the Sensitivity Conjecture, a celebrated open problem in theoretical computer science. He has been awarded a 2020 Alfred P. Sloan Research Fellowship.