


复旦大学数学科学学院
数学综合报告会

报告题目：Isothermal coordinates on surfaces with a square-integrable second fundamental form. Existence and counterexamples

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报告摘要：

The question of the existence of isothermal coordinates on two-dimensional surfaces goes back to the work of Gauss on differential geometry. The first nonlocal theorem on the existence of isothermal coordinates for quadratic differential forms with smooth coefficients was proved by Lichtenstein (1916). For forms with bounded coefficients this result was established by Morrey(1938). Morrey's theorem has been repeatedly reproved and refined. In modern literature, it is often referred to as the Ahlfors-Bers-Bojarski-Morrey theorem. Here we should also mention the result of Helein (2002) on the existence isothermal coordinates for differential quadratic forms with Sobolev coefficients.

For many applications, it is important to find isothermal coordinates with a uniformly bounded conformal factor logarithm. Such coordinates are called bi-Lipschitz coordinates. The existence of bi-Lipschitz coordinates is an essential ingredient of the mathematical theory of biological membranes.In 1994 Toro proved the remarkable theorem on the existence of bi-Lipschitz isothermal coordinates for surfaces with a square-integrable second fundamental form.It should be noted that her approach is based on the theory of varifolds and geometric measure theory. An analytical approach to the problem was proposed in the works of Kuwert andLi, and Riviera (2012). They introduced a class of weak immersions with a square-integrable second fundamental form. An immersion of a two-dimensional closed manifold into a Euclidean space belongs to this class if its first fundamental form is uniformly bounded above and below, and its second fundamental form is square integrable.

A common belief is the existence of bi-Lipschitz isothermal coordinates for all such immersions. This fact is widely used in the mathematical theory of biological membranes. In the proposed work, we show that this assertion is not true in the general case. We give an example of weak immersion of a two-dimensional sphere for which there are no bi-Lipschitz isothermal coordinates. On the other hand, we prove the existence of such coordinates for all weak immersions of tori. The connection of these results with Teichmüller's theory is discussed. Our approach is based on the Chern-Helein moving frame method and the Moser-Struwe result on the validity of the Liouville theorem for elliptic equations with bounded periodic coefficients.

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